The Role of Detector in Which-Way Experiment

Zheng-Chuan Wang
Department of Physics, The Graduate School of the Chinese Academy of Sciences, P. O. Box 4588, Beijing 100049, China.

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Abstract

Following Scully et al.′s study on the mechanism of complementarity, we further investigate the role of detector in which-way experiment. We will show that the initial quantum pure state of particle will reduce to a mixture state because of the inevitable interaction between particle and detector, then the coherence of wavefunction for the particle falling on the screen will be destroyed, which leads to the disappearance of interference fringes in which-way experiment.

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Both Bohr′s principle of complementarity[1] and Feynman′s two-slit experiment [2] manifest the ′wave-particle duality of matter′, in which the loss of interference fringes constitute a mystery of quantum mechanics. There exist controversial explanations on the mechanism of complementarity. In which-way experiment, the disappearance of interference fringes is usually explained by use of the Heissenberg′s uncertainty principle[3]. However, in 1991, Scully et al. performed a quantum optical tests of complementarity [4], and attributed the disappearance of interference fringes to the correlation between the measuring apparatus and the system being observed, not to the usual position-momentum uncertainty principle. Their viewpoints was criticized by Storey et al[5]., the latter still insisted that the uncertainty principle may account for the loss of interference fringe. In 1998, Dürr et al. proposed a which-way experiment to further explore the origin of quantum mechanical complementarity [6] by use of an atom interferometer, they concluded that correlation between the which-way detector and the atomic motion will destroy the interference fringes, and Heissenberg′s position-momentum uncertainty principle can not explain the loss of interference fringes. There are other experiments, such as the atom interferometer experiment by Chapman et al.[7], the electron double-path interferometer experiment by Buks et al.[8], concering with this scheme, too. In this paper, we will further elucidate the disappearance of interference fringes based on the mechanism proposed by Scully et al.

We now consider a which-way experiment, in which the wavefunction describing the center-of-mass motion of particle corresponding to the two slits are $\Psi_1(r)$ and $\Psi_2(r)$, respectively. When we make use of an arbitrary detector to

determine the path of a particle through a fixed double slit, the interaction between particle and detector occurs, which makes the state vectors of particle and detector become entangled. The quantum state of the combined system of particle-detector evolves as follows [9]

$$\begin{aligned} |\Phi(t &= t_0)\rangle = |D_0\rangle \otimes |\Psi(r)\rangle \\ &\longrightarrow \quad |\Phi(t > t_1)\rangle = C_1 a|D_1\rangle|\Psi_1(r)\rangle + C_2 b|D_2\rangle|\Psi_2(r)\rangle. \end{aligned} \tag{1}$$

In the above, $|D_0\rangle$, $|D_1\rangle$, $|D_2\rangle$ are the state vectors of detector, while $|\Psi(r)\rangle = a|\Psi_1(r)\rangle + b|\Psi_2(r)\rangle$ describes the quantum state of particle before being detected. As a result of particle-detector interaction, the correlation between particle and detector has been established after time t_1, the state vectors of particle and detector have coupled to each other after time t_1. Expression (1) clearly demonstrates the violation of pure state $|\Psi(r)\rangle$ after being detected by a detector to determine the path of particle. We can show this violation by its density matrix, too. The reduced density matrix of particle is

$$\begin{aligned} \rho_3 &= Tr_D[|\Phi(t > t_1)\rangle\langle\Phi(t > t_1)|] \\ &= (|C_1 a|^2 + |C_1 a|^2|\langle D_1|D_2\rangle|^2)|\Psi_1(r)\rangle\langle\Psi_1(r)| \\ &\quad +(2C_1 C_2^* ab^*\langle D_2|D_1\rangle)|\Psi_1(r)\rangle\langle\Psi_2(r)| \\ &\quad +(2C_2 C_1^* ba^*\langle D_1|D_2\rangle)|\Psi_2(r)\rangle\langle\Psi_1(r)| + \\ &\quad (|C_2 b|^2 + |C_2 b|^2|\langle D_1|D_2\rangle|^2)|\Psi_2(r)\rangle\langle\Psi_2(r)|, \end{aligned} \tag{2}$$

above Tr_D indicates partial trace over the detector degrees of freedom. When the state vectors $|D_1\rangle$, $|D_2\rangle$ of detector are orthogonal to each other, the density matrix can reduce to

$$\rho_3 = |C_1 a|^2|\Psi_1(r)\rangle\langle\Psi_1(r)| + |C_2 b|^2|\Psi_2(r)\rangle\langle\Psi_2(r)|, \tag{3}$$

which indicates pure state $|\Psi(r)\rangle$ has become to a mixture state. Generally, the pure state $|\Psi(r)\rangle$ of particle will reduce to a mixture state after being detected by the detector, in the end the particle is not in the pure state but a mixture state when it arrives at the screen. The interference fringes will disappear because of the decoherence of the pure state of particle after being detected.

In the general, there exists deviation between the mixture state of particle after interacting with detector and the initial quantum pure state $|\Psi(r)\rangle$. We can evaluate the above deviation by the difference between ρ_3 and the density matrix ρ_1 of pure state $|\Psi(r)\rangle = a|\Psi_1(r)\rangle + b|\Psi_2(r)\rangle$, it is

$$\delta = \sqrt{\sum_{n,m} |(\rho_3)_{nm} - (\rho_1)_{nm}|^2}. \tag{4}$$

Considering the density matrix ρ_3 in the above expression (2), this deviation can be further written as

$$\delta^2 = ||C_1a|^2 + |C_1a|^2|\langle D_1|D_2\rangle|^2 - |a|^2|^2 + |2C_1C_2^*ab^*\langle D_2|D_1\rangle - ab^*|^2 \quad (5)$$
$$+|2C_2C_1^*ba^*\langle D_1|D_2\rangle - ba^*|^2 + ||C_2b|^2 + |C_2b|^2|\langle D_1|D_2\rangle|^2 - |b|^2|^2.$$

We can see that the deviation is determined by the state vectors of detector. If we properly chose the detector and make the state vector $|D_1\rangle = |D_2\rangle$, and the coefficient $C_1 = C_2 = \frac{1}{\sqrt{2}}$, then the deviation will vanish. In this special case, the state vector of combined particle-detector system is $|\Phi(t > t_1)\rangle = \frac{1}{\sqrt{2}}|D_1\rangle \otimes (a|\Psi_1(r)\rangle + b|\Psi_2(r)\rangle)$, there is no correlation between state vectors of particle and detector at all, the quantum state of particle still remain in a pure state after this special measurement by detector. In this case, the interference fringes will not disappear. However, the detector will not distinguish the path of particle because there are no correlation between particle and detector.

If there are no correlation between particle and detector, the wavefunction of particle in the interference region is

$$\Psi(r) = \frac{1}{\sqrt{2}}[\Psi_1(r) + \Psi_2(r)], \quad (6)$$

and the probability density of particle falling on the screen is

$$P(R) = \frac{1}{2}[|\Psi_1(R)|^2 + |\Psi_2(R)|^2 + \Psi_1(r)^*\Psi_2(r) + \Psi_2(r)^*\Psi_1(r)]. \quad (7)$$

When we want to determine the path of particle, the correlation will inevitable occur, we should write the wavefunction of the combined particle-detector system as expression (1). However, the probability density at the screen can not be written as

$$P(R) = \frac{1}{2}[|\Psi_1(R)|^2 + |\Psi_2(R)|^2 + \Psi_1(r)^*\Psi_2(r)\langle D_1|D_2\rangle + \Psi_2(r)^*\Psi_1(r)\langle D_2|D_1\rangle]. \quad (8)$$

Because the interference fringes originate from the particle not the detector, only the particle can fall on the screen, while the detector can not, so the state vectors $|D_1\rangle$, $|D_2\rangle$ of detector can not appear in the expression of probability density at the screen, we can not merely judge the disappearance of interference fringes by the factors $\langle D_1|D_2\rangle$ and $\langle D_2|D_1\rangle$ in expression (8). In fact, generally, the particle is in a mixture state after detected by the detector, the more precise description of the disappearance of interference fringes should be based on expression (5), this disappearance is determined not only by the factors $\langle D_1|D_2\rangle$, $\langle D_2|D_1\rangle$, but also by the coefficients C_0, C_1. Only in the special case of $C_0 = C_1 = \frac{1}{\sqrt{2}}$, the disappearance of interference fringe is determined by $\langle D_1|D_2\rangle$, $\langle D_2|D_1\rangle$.

In Scully et al.'s experiment, the detector are two maser cavity systems, the state vectors of detector are described as $|1_10_2\rangle$ and $|0_11_2\rangle$, where $|1_10_2\rangle$ denotes the state in which there is one photon in cavity 1 and none in cavity

2, the interaction between atom beam and maser cavity system lead to the correlation between them, and the initial pure state of atom will reduce to a mixture state when it arrives at the screen, which causes the disappearance of interference fringes. In dürr et al.$'$s experiment, two internal electronic states $|2\rangle$ and $|3\rangle$ of $^{85}R_b$ atom are used as a which-way detector system. Since the states of detector and the states of center-of-mass motion belong to the same atom, both of them can appear on the screen, the state vectors $|2\rangle$ and $|3\rangle$ of detector can appear in the probability density at the screen similar to (8), then the loss of interference fringes is determined by the factors $\langle 2|3\rangle$ and $\langle 3|2\rangle$. However, as pointed out by dürr et al., there are additional states of detector must be considered except the internal electron states of atom, they are the quantum states $|\alpha\rangle$ and $|\beta\rangle$ of microwave field, where $|\alpha\rangle$ denotes the initial state of microwave field, $|\beta\rangle$ is the quantum state after the absorption of one photon. $|\alpha\rangle$ and $|\beta\rangle$ can not appear on the screen hence in the expression of probability density, the complete states of detector should be $|2\rangle|\beta\rangle$ and $|3\rangle|\alpha\rangle$, not merely the internal electronic states $|2\rangle$ and $|3\rangle$ of atom, so in essence, we also need discuss the disappearance of interference fringes by use of formula (5). In Dürr et al.$'$s experiment, the entanglement of the atom with the microwave field can be simply neglected because the initial state $|\alpha\rangle$ is a coherent state with a large mean photon number and a large spread of the photon number, so we can approximately discuss the disappearance of interference fringes by expression (8), otherwise, we must study this issue by expression (5).

In summary, we have further shown the role of detector in which-way experiment. It is this interaction between particle and detector that leads to the change of quantum state of particle from initial pure state into a mixture state, and the disappearance of interference fringes. We also describe the disappearance of interference fringes by a deviation between the initial pure state and the mixture state of particle, which is consistent with the experimental results of Schully et al$'$s and Dürr et al.$'$s.

Acknowledgments

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